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MINVO Basis: Finding simplexes with minimum volume enclosing polynomial curves

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Abstract-

This paper studies the polynomial basis that generates the smallest n-simplex enclosing a given nth-degree polynomial curve in Rn. Although the Bernstein and B-Spline polynomial bases provide feasible solutions to this problem, the simplexes obtained by these bases are not the smallest possible, which leads to overly conservative results in many CAD (computer-aided design) applications. We first prove that the polynomial basis that solves this problem (MINVO basis) also solves for the nth-degree polynomial curve with largest convex hull enclosed in a given n-simplex. Then, we present a formulation that is independent of the n-simplex or nth-degree polynomial curve given. By using Sum-Of-Squares (SOS) programming, branch and bound, and moment relaxations, we obtain high-quality feasible solutions for any n∈N, and prove (numerical) global optimality for n=1,2,3 and (numerical) local optimality for n=4. The results obtained for n=3 show that, for any given 3rd-degree polynomial curve in R3, the MINVO basis is able to obtain an enclosing simplex whose volume is 2.36 and 254.9 times smaller than the ones obtained by the Bernstein and B-Spline bases, respectively. When n=7, these ratios increase to 902.7 and 2.997⋅1021, respectively.

Index Terms- Minimum enclosing simplex; Curve with largest convex hull; Polynomial basis; Polynomial curve; Spline

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